Abstract—Full duplex communication requires nodes to cancel their own interference. Recent work have proved the feasibility of full duplex communications using software radios. In this paper, we address capacity comparisons when the total amount of analog radio hardware is bounded. Under this constraint, it is not immediately clear if one should use these radios to perform full-duplex self-interference cancelation or use the radios to give additional MIMO multiplexing advantage. We find that repurposing radios for cancellation, instead of using all of them for half-duplex over-the-air transmission, can be beneficial since the resulting full-duplex system performs better in some practical SNR regimes and almost always outperforms half duplex in symmetric degrees-of-freedom (large SNR regime).

I. INTRODUCTION

Recent measurement based studies, comparing $2 \times 2$ MIMO half-duplex systems with $2 \times 1$ MISO full-duplex systems [1], have demonstrated that full-duplex communication can outperform in many cases. From the point of view of high-SNR multiplexing gain, both configurations should have identical multiplexing gain of 2. However, measurement-based results suggest that full duplex achieves better ergodic rate as compared to half-duplex, for medium to high signal to noise ratios.

In this paper, we compare full- and half-duplex systems which have similar analog radio hardware resources. Often the number of antennas are limited by a mix of available device area (which limits the number of uncorrelated antennas one can place for MIMO systems) and analog radio resources to drive the antennas (which are generally power hungry). Thus, we compare full- and half-duplex systems which have comparable analog radio resources. In particular, we measure radio resources in the form of sum total of up-converting radio chains (from baseband to RF) and down-converting radio chains (from RF to baseband).

Our analysis is performed in two different SNR regimes: (i) a finite SNR analysis which uses a more detailed signal model to capture the effects seen in actual devices and which have an impact on both full and half-duplex MIMO systems, and (ii) an asymptotic-in-SNR analysis to find the degrees of freedom (or multiplexing gain) of both systems using the more traditional signal model.

Experiment based analysis have studied the factors that dominate the performance of full- and half-duplex MIMO systems. For full-duplex systems it has been observed by all the reported methods for self-interference cancellation cannot suppress the self-interference down to the noise floor [1–3, and see references within]. As a result the residual self-interference raises the noise floor and is a dominant factor in the performance of full-duplex systems. Further, for MIMO systems it has been observed that the condition numbers are seldom like those predicted by i.i.d. Rayleigh channel model [4], which leads to a loss in capacity of MIMO systems.

The key question is how do the two systems compare at realistic SNRs. Our capacity based numerical comparisons for practical SNRs shows that the full-duplex systems outperform half-duplex in many cases, but not all of them. It is obvious that imperfect cancelation only hurts full-duplex. However, asymmetry in MIMO channels hurts half-duplex systems more than their full-duplex counterparts.

Our degrees of freedom analysis shows that for most cases, full-duplex achieves higher degrees of freedom, even at the expense of using some of its radio resources to cancel self-interference. The use of radio resources to cancel self-interference appears wasteful at first, but by being able to suppress self-interference, bi-directional data streams can be simultaneously supported, which leads to overall gain in the symmetric degrees of freedom. We note that imperfect self-interference cancellation amounts to a higher noise floor which does not affect the degrees of freedom results.

The rest of the paper is organized as follows. In Section II, we provide the channel model that is used in the paper. Section III describes a self-interference cancellation method which uses an additional transmitter radio to generate a cancellation signal which forms the basis of this paper. Section IV describes our results in the practical signal to noise ratio regimes while Section V describes our results in high signal to noise ratio regimes.

II. CHANNEL MODEL

We consider a two-node system, where the two nodes are communicating with each other. The two nodes are denoted by $N_1$ and $N_2$. For accurate accounting for radio hardware resources, we will distinguish between up-converting and down-converting hardware, instead of treating it as one monolithic radio performing both up- and down-conversion. Implicit in our accounting is the assumption that an up-converting chain...
consumes same hardware resources as a down-converting chain. Thus, we will equate total up- and down-converting radio resources for the full- and half-duplex systems.

Assume that node \( N_i \) has a total of \( A_i \) up- and down-converting radio chains, \( i = 1, 2 \). Out of these \( A_i \) radio chains, \( m_i \) are up-converting used for transmission and \( n_i \) are down-converting used for reception. Often, \( m_i = n_i = \frac{A_i}{2} \) in half-duplex systems.

In the full-duplex implementation (see Section III for more details), \( c_i \) up-converting chains are used for active analog self-interference cancellation. Thus, the number of transmit antennas in our full-duplex analysis is \( m_i - c_i \), which is less than \( m_i \) antennas used in half-duplex counterpart. For the full duplex design in Section III, we assume that \( c_i = n_i \).

The input-output relation is given by

\[
N_i \rightarrow N_j : Y_j = H_{ij}X_i + W_j, \quad (1)
\]

where the elements of \( W_j \) are assumed to be i.i.d. with complex normal distribution of zero mean and variance, \( CN(0, \sigma_i^{2}_{FD/HD}) \). Since self-interference cancellation in practical systems is not perfect, the effective noise floor of current generation of full-duplex implementations is higher than half-duplex counterparts, which implies \( \sigma_i^{2}_{FD} > \sigma_i^{2}_{HD} \).

The dimension of \( H_{ij} \) is \( m_i \times n_j \) for half-duplex and \( (m_i - c_i) \times n_j \) for full-duplex. Further, elements of \( H_{ij} \) and \( H_{ji} \) are uncorrelated and independent of each other, but not necessarily i.i.d. We will consider a special form of non-i.i.d. distribution, caused by different amount of path loss between different transmit-receive pairs in MIMO systems, since it is more general and representative of the channel conditions observed in deployed systems.

We also assume that the sum power used by the nodes in each timeslot is SNR. For full-duplex, each node uses a power of \( \text{SNR}/2 \) while power of SNR is used in half-duplex mode. We further assume that \( T_{coh} \) is coherence interval such that the channel \( H \) is fixed during a fading block of \( T_{coh} \) consecutive channel uses, and statistically independent from one block to another. We further assume that \( T_{coh} \) is finite and does not scale with SNR.

III. FULL DUPLEX SYSTEM COMPONENTS

In this paper, we consider a full duplex system proposed in [1, 2] where an additional up-converting radio circuit is used for self-interference cancellation at the carrier frequency. The system naturally scales well for MIMO and wideband systems and thus will be focus of our analysis in this paper.

The full-duplex system uses a combination of passive suppression and active cancellation techniques, where passive suppression precedes active cancellation. The cancellation techniques are explained below.

**Passive Suppression (PS):** This suppression is achieved by maximizing the attenuation of the self-interference signal due to propagation path loss over self-interference channel, which is the channel between the transmitter and receiver antenna on the same node. The amount of passive suppression depends on the distance between antennas, the antenna directionality, and the antenna placement on the full-duplex device.

**Active Analog Cancellation (AC):** As the name suggests, this is the active cancellation performed in analog domain before the received signal passes through the analog-to-digital converter.

![Fig. 1. Full Duplex 2×1 node. The gray boxes depict analog cancellation.](image)

The hardware components required for active analog cancellation of the self-interference at one receiver antenna consist of one digital-to-analog converter, one transmitter radio which up converts the signal from base-band to center frequency, and one analog adder. Figure 1 shows the example block diagram of analog cancellation for the full-duplex node with two transmitter antennas and one receiver antenna. One input to the analog adder is the signal at the receiver antenna and the other input is a canceling signal local to node \( i \) communicated via a wire.

It is important to notice that the additional hardware requirements of the analog canceller scales linearly only with the number of receive antennas and is independent of the number of transmit antennas (i.e. number of canceller radios is the same as the number of receiver radios). Another important aspect to notice is that our analog cancellation does not impose any constraint on the design of transmitted signals. As expected, the analog cancellation is not perfect and thus an additional digital cancellation was also used in [1].

**Digital Cancellation (DC):** There is a residual self-interference that remains after analog cancellation due to imperfect analog cancellation. Active digital cancellation subtracts the estimated interference from the received signal in the digital domain. The estimate of interference is computed based on a second round of pilots sent from each transmitter antenna and received while applying analog cancellation to each receiver antenna. Alternatively, the estimate of interference can be computed without extra pilots if implemented based on correlation between the transmitted and received self-interference payload signal.

As an example implementation in [1] for WiFi-like systems, the above approach of cancellation can achieve a median cancellation of 85 dB. Hence, using the above approach of self-interference cancellation can reduce the self interference...
floor to 10 dB if the noise floor of $-90$ dBm (typical of WiFi cards). In this paper, we compare full-duplex and half-duplex in both practical and high SNR regimes. For our high SNR regime analysis, we assume perfect self-interference cancellation and are inspired by our understanding that better radio technology could potentially push self-interference down to noise floor. The high SNR analysis provides some insights about practical SNR since the cross-over SNR for the two schemes is dependent on the difference in slopes at high SNR. To complement the high SNR analysis, we provide numerical results in medium SNR to observe the effects of imperfections in the system, which are more dominant at practical SNRs.

IV. PRACTICAL SNR COMPARISON

For the case of i.i.d. channel matrix $H$ and the additive noise $W$, the optimal MIMO transmission uses circularly symmetric codes [5]. However, if either $H$ or $W$ is not i.i.d Gaussian, then optimal codes can leverage the channel correlations. However, the optimal codes require knowledge of the channel statistics, which may or may not be known accurately. This is one of the primary reasons that none of the wireless standards try to optimize transmit covariance matrix. Thus, we will assume a transmit covariance of identity and assume no power control. Further, we assume that the training for the other node is perfect and there is no channel estimation error (essentially this section results implicitly assume large coherence interval for simplicity). Thus, the achievable symmetric rates for half-duplex and full-duplex are given by

$$R_{HD} = \frac{1}{2} \mathbb{E}(\log \text{det}(I + SNR H_{HD} H_{HD}^\dagger)),$$

and

$$R_{FD} = \log \mathbb{E}(\text{det}(I + SNR/2 H_{FD} H_{FD}^\dagger/(1 + \text{INR})))$$

respectively. Here, $H_{HD}$ and $H_{FD}$ are the half- and full-duplex channel matrices, INR represents the residual self-interference relative to the noise level, and $\mathbb{E}$ represents the expectation with respect to the statistics of the channel matrices.

Since a closed form evaluation is not possible, we will use numerical results to compare the relative performance of full- and half-duplex at practical SNRs.

We use the example of $A_1 = A_2 = 6$ radios. This allows a $2 \times 2$ full-duplex ($m_1 = 4, c_1 = 2, n_1 = 2$) or a $3 \times 3$ half-duplex ($m_1 = n_1 = 3$). As shown in Section V, for $A_1 = A_2 = 6$ radios, the symmetric degrees of freedom for full duplex is 2, while for half-duplex the symmetric dof is 3/2. This is because for a full duplex system, 2 x 2 system can be used in both directions simultaneously while for the half-duplex system, 3 x 3 configuration is possible for each node but only half the time. Thus, at large enough SNR, full duplex will give better rate than half-duplex system.

We first consider only the effect of imperfect self-interference cancellation and model $H$ as i.i.d. If the self-interference cannot be cancelled completely, it results in an elevated noise floor (of $1+\text{INR}$ times higher power than noise).

The increased noise floor immediately implies a performance degradation for full-duplex system at finite SNR. We see in Figure 2 that imperfect cancellation determines the SNR before which full duplex becomes better than half duplex. The crossover point where full duplex performs better than half-duplex increases with the amount of residual self-interference cancellation. Full Duplex performs better than half-duplex at SNR above about 25dB, 39dB, 54dB for self-interference 0dB, 5dB, 10dB above noise floor respectively.

The second effect that we observe in deployed systems is that the elements in the channel matrix $H$ do not have the same variance. The non-identical variance occurs due to device orientation causing different transmit-receive antennas to have different path-losses. While non-identical variances do not reduce the degrees of freedom (since the ratio of variances of different channel coefficients is independent of SNR), it does affect the performance at finite SNR.

We model channel variance asymmetry as follows. For a $n \times m$ channel matrix, all rows from 2 to $n$ have elements $\mathcal{CN}(0, a)$ for some $a < 1$. The variation in $a$ produces varying results on HD and FD. Assuming self-interference at the noise floor, Figure 3 shows the affect of asymmetry on the rate for $A_1 = A_2 = 6$. We note that asymmetry decreases the rate of both full- and half duplex systems. We also note that asymmetry affects half-duplex more since two of the three paths are impacted while one out of two paths is impacted for the full-duplex system. Consequently, with more asymmetry, the crossover SNR point, where full-duplex starts performing better than half-duplex, decreases. For example, the cross over point decreases from about 25 dB to about 10dB when the asymmetry goes from from 0 dB to $-10$ dB.

V. LARGE SNR COMPARISON

In this section, we compute the symmetric degrees of freedom, which is defined as the ratio of symmetric rate to $\log(SNR)$ as SNR goes to infinity. Symmetric degrees of
freedom is the minimum of the degrees of freedom from one node to another. We assume that training is performed so that each receiver can learn the channel and that the estimation error is considered as noise. We note that we will assume perfect self-interference cancellation (since imperfect cancellation is modelled as a raised noise floor and does not affect the analysis at high signal to noise powers) and further assume i.i.d. channel matrices.

**A. Degrees of freedom for half-duplex**

For the half-duplex system, we assume that $N_1$ transmits half the time and $N_2$ transmits the other half of the time. The degrees of freedom for the direction from $N_1$ to $N_2$ is

$$d_{HD} = \max_{m_1, m_2 \leq A_i} \frac{1}{2} \min \left( \frac{T_{coh} - m_1}{T_{coh}}, \frac{T_{coh} - m_2}{T_{coh}}, \frac{T_{coh} - m_1}{T_{coh}} \right).$$

For half-duplex operation, no canceller antennas are needed. Thus, the optimal degrees of freedom is given by the following theorem.

**Theorem 1.** The optimal symmetric degrees of freedom for half-duplex with each node $i = 1, 2$ with $A_i$ up/down-converting radio chains is given by,

$$d_{HD}^i = \frac{1}{2} \min \left( \frac{T_{coh} - m_1}{T_{coh}}, \frac{T_{coh} - m_2}{T_{coh}} \right).$$

**Remark 1.** As $T_{coh} \to \infty$, $d_{HD} = 1/2\min(A_1, A_2)/2$. This is achieved by $m_1 = \lfloor \min(A_1, A_2)/2 \rfloor$.

**B. Degrees of freedom for full-duplex**

For a full-duplex implementation at high SNR, we ignore the digital cancellation used and hence calculate the training time used up only due to analog cancellation. The training symbols sent by the transmitter antennas is used both by the same node for self-interference cancellation and the other node to get a training on the channel gains.

In the full-duplex system $N_1$ and $N_2$ transmit simultaneously. The degrees of freedom for the direction from $N_1$ to $N_2$ is

$$d_{FD} = \frac{1}{2} \min \left( \frac{T_{coh} - m_1}{T_{coh}}, \frac{T_{coh} - m_2}{T_{coh}}, \frac{T_{coh} - m_1}{T_{coh}} \right).$$

Thus, the optimal degrees of freedom is given by the following theorem.

**Theorem 2.** The optimal symmetric degrees of freedom for full-duplex with each node $i = 1, 2$ with $A_i$ radio chains is given by,

$$d_{FD} = \max_{c_i, m_i \leq A_i} \frac{T_{coh} - m_1}{T_{coh}} \min \left( m_1 - m_2 + c_1 + c_2, \frac{T_{coh} - m_1}{T_{coh}} \right) - \frac{T_{coh} - m_2}{T_{coh}} \min \left( m_1 - c_1, m_2 - c_2 \right).$$

**Remark 2.** As $T_{coh} \to \infty$, $d_{FD} = \min(A_1, A_2)/3$. This is achieved by $c_i = m_i - c_i = \min(A_1, A_2)/3$.

**C. FD or HD?**

We first compare the full-duplex and half-duplex when the coherence time is large enough. We only compare for the case when $\min(A_1, A_2) \geq 2$ since otherwise half-duplex and full-duplex give zero symmetric rate. We find that for $\min(A_1, A_2) = 2$, half-duplex performs better since full-duplex cannot be used while for greater radio resources, full-duplex performs better. The symmetric degrees of freedom is also depicted in Figure 4.

**Theorem 3.** As $T_{coh}$ goes to infinity, $d_{FD} \geq d_{HD}^i$ for $\min(A_1, A_2) > 2$, and $d_{FD} < d_{HD}^i$ for $\min(A_1, A_2) = 2$.

The above shows that it is better to use radios for cancellation than for MIMO gains in half-duplex fashion for $\min(A_1, A_2) > 2$. However, the above analysis did not ac-
count for the fixed coherence time. For a fixed coherence time, full duplex needs more time for training \(2\lfloor \min(A_1, A_2)/3 \rfloor \) as compared to half-duplex \(\lfloor \min(A_1, A_2)/2 \rfloor \). Thus, for a small coherence time, the optimization performed may yield better performance of half-duplex than full-duplex.

VI. Conclusion

In this paper, we compared the performance of half-duplex and full-duplex systems at both practical and high signal to noise ratio regimes. We find that even though full-duplex gives better performance in high signal to noise ratio, we also need to work with the practical imperfections to decide whether or not full duplex should be useful. If the self-interference cancellation is not perfect, full-duplex performs better at much higher signal powers. However, when some of the transmit antennas face higher fading, full-duplex system performs better at lower signal powers.

References