Joint Optimal Power Allocation and Base Station and Relay Station Placement in Wireless Relay Networks

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***Abstract*—In this paper, we consider a finite geographic area with multiple mobile stations (MSs) uniformly distributed within the area and multiple candidate locations (CLs) for deploying base stations (BSs) and relay stations (RSs) to serve the MSs. For this network scenario, we study the joint optimal placement of BSs and RSs into those CLs and MS and RS power allocations such that the sum-capacity of the network is maximized while the target data rate of each MS is achieved. In order to investigate the energy-efficiency trade-offs between deploying BSs and RSs, we provide an iterative algorithm which first maximizes the sum-rate of the network by optimally deploying a certain number of BSs. Then, the algorithm decreases the number of BSs to be deployed optimally by one and continues deploying RSs until the same sum-rate is achieved. The process continues until the number of optimally deployed BS is 1 and the number of optimally deployed RSs is less than or equal to the total number of candidate RS locations. Our numerical results suggest that significant gains in terms of reduction of total transmitted power can be obtained by replacing BSs with RSs. However, this gain diminishes when the number of BSs became too small which makes the BS-RS and RS-MS distances too large for energy efficient communications.**

I. INTRODUCTION

In the face of increasing worldwide energy demand, global warming concerns, and volatility in energy supplies and prices, governments, standards bodies, the research community, and industry have all recently recognized the need to improve the energy efficiency of the communications part of the information and communication technology (ICT) sector. Within the communications component of ICT, recent studies have shown that the largest elements of power consumption are access networks (wired and wireless) rather than core networks. This is not surprising due to the larger number of network components in access networks compared to core networks and the pervasiveness of relatively power efficient optical technology in the core. Reducing the total power consumption of wireless networks is is an well recognized way to improve the energy efficiency and thus to contribute to the reduction of world-wide energy consumption.

Given the clear need to reduce the energy consumption, the fundamental challenge is how to reduce the overall power consumption of wireless networks while maintaining adequate coverage, quality of services, and reliability. Wireless relay network (WRN) can provide a favorable platform to address this challenge. The underlying technology of WRN is cooperative communications (CC), which is shown to be a promising approach to increase data rates and reliability in wireless networks [1]–[3]. In WRNs, lower energy consumption is achieved via using less transmission power due to smaller distances between relays and terminals, spatial diversity, and using efficient signal processing schemes such as distributed beamforming [4], distributed space-time coding [5], [6], etc. Replacing long-distance MS-BS links with multiple shorter MS-RS and RS-BS links results in an enhanced end-to-end data rate since the links with shorter distances suffer less attenuation. Proper placement of RSs may also improve the channel quality by removing any obstacles between MSs and BSs. Another attractive feature of RSs is that RSs are less complex and have lower installation and maintenance costs than those of BSs. Due to these appealing features of RSs, WRNs have drawn extensive attention among both industry and academia. It should be noted that an efficient BS and RS placement strategy along with the optimal assignment of BSs and RSs with MSs, and optimal MS and RS power allocations are critical in exploiting the full benefits of WRNs.

Regardless of the numerous advantages of RSs, one cannot ignore the fact that RSs have smaller coverage and less resources (e.g. transmission power or bandwidth) as compared to those of BSs [7], which indicates a trade-off between the cost and efficiency of BSs and RSs in the joint optimal BS and RS placement and MS and RS power allocations problem. In our joint optimal power allocations across MS and RS transmissions and BS and RS placement problem, we investigate the trade-off in energy efficiency of BSs and RSs in achieving a certain sum-capacity of the network. Specifically, we consider a finite geographic area with multiple BS and RS candidate locations. A certain numbers of BSs and RSs have to be placed in those candidate locations to serve a number of MSs uniformly distributed within the geographic area. The objective is to optimally place BSs and RSs into candidate locations, assign them optimally with MSs, and allocate optimal powers among MS and RS transmissions such that the sum-capacity of the network is maximized while the target rate of each MS is achieved. We note that this problem formulation involves integer variables (to characterize RS and BS assignment with MS) and nonlinear constraints (target data rate constraints). It is well known that in general, a mixed-integer nonlinear program (MINLP) is NP-hard, which is the main difficulty here. However, an MINLP formulation does not mean the problem itself is NP-hard (unless the problem is proved to be NP-hard). We use TOMLAB [8], a MATLAB based commercial software, to find the near optimal solution of the combinatorial problem.

In order to investigate the energy-efficiency trade-off between BSs and RSs, we first consider a BS-only deployment scenario and solve the joint optimization problem for a certain number of BSs and no RS. The maximum capacity achieved by this deployment is treated as a target sum-capacity for subsequent relay based deployments, where in each case, we reduce the number of BSs to be placed optimally by 1 and increase the number of RS placements to solve the joint optimization problem until the target sum-capacity is achieved. For each case, the total MS plus RS transmission power is calculated and compared.

A. Related Work

The node placement problem in telecommunication domain, which targets at optimal placement of network equipment such as BSs, access points (APs), RSs, and gateways, is extended from the fundamental study of facility location and k-median problem in operations research. Due to the variety of practical placement in wireless networks, we have witnessed a great diversity in the design objectives of network architecture and placement algorithms. Recently, RS placement problem has drawn great attention in different application scenarios, such as cellular networks [9], wireless sensor networks [10]–[12], wireless local area networks [13], and IEEE 802.16j networks [7], [14]–[18]. In [13], Zou *et al.* studied the placement of a given number of relay stations in a multi-rate WLAN cell with a given MS distribution. They solved the optimization problem by an iterative algorithm based on Lagrangian relaxation. While the objective of the work in [11] involve the efforts in optimizing network lifetime, the objective of the work in [13] is to improve the network throughput. However, both objectives create cost-effective carrier environment with on-demand resource allocation. Hou *et al.* [12] formulated the joint energy provisioning and relay node placement problem into an MINLP, and a heuristic was developed to solve it.

Lin *et al.* considered the problem of optimal RS placement for a single RS [14] and for two RSs [15] in an 802.16j type cell using decode and forward (DF) cooperative relaying technology. In [16], [17], the authors developed an integer programming formulation for the problem of realizing coverage in relay architectures. Unlike the work in [16], [17], in their subsequent work, Yu *et al.* considered the capacity constraints at BSs in joint BS and RS location planning. In [7], the authors considered jointly deploying a number of BSs and RSs to serve MSs distributed arbitrarily in a given geographic area such that the cost is within a predefined budget and the system capacity is maximized. The authors in [7] investigate the impact of different deployment profiles on network capacity and fairness and cost-efficiency trade-offs between BSs and RSs. Their numerical results show that the deployment of RSs results in enhanced system capacity and fairness as long as the number of BSs is not too small. Finally, in [19], the authors study the impact of relaying on the coverage of 3GPP LTE-Advanced. They propose a novel evaluation methodology that can be used to find a relation of relay node (RN) transmission power, ratio between number of BSs and RNs, and performance of the system.

All the above works considered fixed transmission powers by BSs (for downlink), RSs (for both downlink and uplink) and MSs (for uplink). However, power control is necessary to reduce the energy consumption and optimal assignment of BSs and RSs to MSs. In this work, we consider joint optimization of MS and RS power allocation, BS and RS placement and assignment of deployed BSs and RSs to MSs.

The rest of the paper is organized as follows. Section II provides the system and signal model. The capacity maximization problem is formulated and analyzed in Section III. In Section IV, we provide an iterative algorithm to investigate the energy-efficiency trade-offs between BSs and RSs. Results of the simulations are provided in Section V. Finally, conclusions are drawn in Section VI.

II. SYSTEM MODEL

We consider a square service area where multiple MSs are distributed within the service area. Within the same area, there are predefined sets of candidate locations for the placement of BSs and RSs. Multiple BSs and RSs can be deployed in the network. An MS can either directly transmit to a BS or transmit to a BS with the assistance of an RS. We assume that if an MS directly transmits to a BS, it does not need the assistance of an RS. Similarly, if an RS assists an MS to forward its data to a particular BS, there would be no direct transmission between that MS and any BS. However, a BS or an RS can be accessed simultaneously by different MSs at their assigned frequency bands using Orthogonal Frequency-Division Multiple Access (OFDMA) technique. In other words, orthogonal transmissions are used for simultaneous transmissions among different MSs by using different channels and time division multiplexing is employed by the relaying schemes. We assume a conventional two-stage amplify-and-forward (AF) relaying scheme [1], [20]. Due to the same orthogonal assumption, multiple RSs can forward the data of multiple MSs to a single BS. We also assume that an MS can be assisted by at most one RS.

Let $\mathcal{S}_{\text{BS}} = \{1, \ldots, N\}$, $\mathcal{S}_{\text{RS}} = \{1, \ldots, M\}$, and $\mathcal{S}_{\text{MS}} = \{1, \ldots, K\}$ be the sets of candidate locations for BS and RS, and the set of MSs, respectively. Sets $\mathcal{S}_{\text{BS}}$, $\mathcal{S}_{\text{RS}}$, and $\mathcal{S}_{\text{MS}}$ are disjoint sets. To keep the description simple, we will use MS$_k$ to denote kth MS and BS$_n$ (RS$_m$) to denote the BS (RS) deployed in the candidate location $n \in \mathcal{S}_{\text{BS}}$ ($m \in \mathcal{S}_{\text{RS}}$). We will also use the notations CL_n^{BS} (CL_m^{RS}) to denote the nth (mth) candidate location of BS (RS). Let g_{kn}, h_{km}, and d_{mn} denote the channel gains for the links MS$_k$-BS$_n$, MS$_k$-RS$_m$, and RS$_m$-BS$_n$, respectively. The channel gains may include the effects of path loss, shadowing, and fading. Flat fading channels are assumed among MS-BS, MS-RS, and RS-BS links. We assume that the transmission frame length is small compared to the channel coherence time and as a result all channel gains can be considered fixed during the time of interest. To facilitate further discussion, we define the following notations.

- P_{kn} : transmit power of MS$_k$, $k \in \mathcal{S}_{\text{MS}}$ directly connected with BS$_n$, $n \in \mathcal{S}_{\text{BS}}$.
- Q_{km} : transmit power of MS$_k$, $k \in \mathcal{S}_{\text{MS}}$ connected with RS$_m$, $m \in \mathcal{S}_{\text{RS}}$.
- $F_{mn}^{(k)}$: transmit power of RS$_m$, $m \in \mathcal{S}_{\text{RS}}$ connected with both MS$_k$, $k \in \mathcal{S}_{\text{MS}}$ and BS$_n$, $n \in \mathcal{S}_{\text{BS}}$.
- b_n : BS location index. b_n = 1 if a BS is placed at CL_n^{BS}; otherwise $b_n = 0$.
- r_m : RS location index. r_m = 1 if an RS is placed at CL_m^{RS}; otherwise $r_m = 0$.
- x_{kn} : location allocation index. $x_{kn} = 1$ if MS$_k$, $k \in \mathcal{S}_{\text{MS}}$ directly transmits to BS$_n$, $n \in \mathcal{S}_{\text{BS}}$; otherwise, $x_{kn} = 0$.
- y_{km} : location allocation index. $y_{km} = 1$ if MS$_k$, $k \in \mathcal{S}_{\text{MS}}$ is assisted by RS$_m$, $m \in \mathcal{S}_{\text{RS}}$; otherwise, $y_{km} = 0$.
- $z_{mn}^{(k)}$: location allocation index. $z_{mn}^{(k)} = 1$ if RS$_m$, $m \in \mathcal{S}_{\text{RS}}$, assists MS$_k$, $k \in \mathcal{S}_{\text{MS}}$ to transmits its data to BS$_n$, $n \in \mathcal{S}_{\text{BS}}$; otherwise, $z_{mn}^{(k)} = 0$.
- c_k : achieved data rate of MS$_k$, $k \in \mathcal{S}_{\text{MS}}$.
- $c_k^{\min}$: minimum data rate demand of MS$_k$, $k \in \mathcal{S}_{\text{MS}}$.
- σ_n^2 : background noise variance at BS$_n$, $n \in \mathcal{S}_{\text{BS}}$.
- ν_m^2 : background noise variance at RS$_m$, $m \in \mathcal{S}_{\text{RS}}$.

If an MS is directly connected with a BS, it needs only one time slot to transmit its signal. On the other hand, if an MS is connected to a BS through an RS, then under AF scheme, in the first time slot, an MS transmits unit power signal to an RS. In the subsequent time slot, assuming the RS knows the channel state information (CSI) for theMS-RS link, the RS normalizes the received signal and retransmits it to the destination BS. Let s_k be the unit power signal, i.e. $\mathbb{E}[\|s_k\|^2] = 1$, transmitted by MS$_k$, where $\mathbb{E}[\cdot]$ denotes the expectation operator. If MS$_k$ is directly connected with BS$_n$, i.e. $x_{kn} = 1$, then the received signal of MS$_k$ at BS$_n$ can be expressed as

$$r_{kn}^{\text{MS}-\text{BS}} = \sqrt{P_{kn}} g_{kn} s_k + u_n \,, \tag{1}$$

where u_n is the additive white Gaussian noise (AWGN) at BS$_n$ with variance σ_n^2. In this case, the maximum achieved capacity of MS$_k$ is given by

$$c_{kn}^{\text{MS}-\text{BS}} = \log_2\left(1 + \text{SNR}_{kn}^{\text{MS}-\text{BS}}\right) \,, \tag{2}$$

where, without the loss of generality, we assume the channel bandwidth to be 1 and

$$\text{SNR}_{kn}^{\text{MS}-\text{BS}} = \frac{P_{kn}|g_{kn}|^2}{\sigma_n^2} \tag{3}$$

is the signal to noise ratio (SNR) at the receiver of BS$_n$ for the direct transmission of MS$_k$.

On the other hand, if MS$_k$ is assisted by RS$_m$ to transmit its data to BS$_n$, i.e. $y_{km} = 1$ and $z_{mn}^{(k)} = 1$, during the first time slot, MS$_k$ transmits s_k to RS$_m$. The signal received by RS$_m$ during the first time slot can be written as

$$r_{km}^{\text{MS}-\text{RS}} = \sqrt{Q_{km}} h_{km} s_k + v_m \,, \tag{4}$$

where v_m is the AWGN at RS$_m$ with variance ν_m^2.

During the second time slot, RS$_m$ normalizes its received signal and retransmits it to BS$_n$. For RS$_m$, the normalization factor is

$$\alpha_{km} = \sqrt{\mathbb{E}[|r_{km}^{\text{MS}-\text{RS}}|^2]} = \sqrt{Q_{km}|h_{km}|^2 + \nu_m^2} \,. \tag{5}$$

Therefore, during the second time slot, the received signal at BS$_n$ can be expressed as

$$\begin{aligned} r_{kmn}^{\text{MS}-\text{RS}-\text{BS}} &= \frac{1}{\alpha_{km}} \sqrt{F_{mn}^{(k)}} d_{mn} r_{km}^{\text{MS}-\text{RS}} + u_n & (6) \\ &= \sqrt{\frac{Q_{km} F_{mn}^{(k)}}{Q_{km}|h_{km}|^2 s_k + \nu_m^2}} d_{mn} h_{km} + \hat{u}_n \,, & (7) \end{aligned}$$

where

$$\hat{u}_n = \frac{\sqrt{F_{mn}} d_{mn}}{Q_{km}|h_{km}|^2 + \nu_m^2} \nu_m + u_n$$

is the equivalent noise term at BS$_n$. It can be easily shown that $\hat{u}_n \sim \mathcal{CN}(0, \hat{\sigma}_n^2)$ with

$$\hat{\sigma}_n^2 = \sigma_n^2 + \frac{F_{mn}^{(k)}|d_{mn}|^2 \nu_m^2}{Q_{km}|h_{km}|^2 + \nu_m^2} \,. \tag{8}$$

BS$_n$ applies maximal ratio combining on the signal received during the second time slots. With AF relaying, the achievable rate of MS$_k$ is given by

$$c_{kmn}^{\text{MS}-\text{RS}-\text{BS}} = \log_2\left(1 + \text{SNR}_{kmn}^{\text{MS}-\text{RS}-\text{BS}}\right) \,, \tag{9}$$

where $\text{SNR}_{kmn}^{\text{MS}-\text{RS}-\text{BS}}$ is the received SNR at BS$_n$ due to the transmission from MS$_k$ via RS$_m$. From (4) and (7), we get

$$\text{SNR}_{kmn}^{\text{MS}-\text{RS}-\text{BS}} = \frac{F_{mn}^{(k)} Q_{km}|d_{mn}|^2|h_{km}|^2}{F_{mn}^{(k)}|d_{mn}|^2\nu_m^2 + \left(Q_{km}|h_{km}|^2 + \nu_m^2\right)\sigma_n^2} \,. \tag{10}$$

Now, since the binary variables $\{x_{kn}\}$, $\{y_{km}\}$, and $\left\{z_{mn}^{(k)}\right\}$ are unknown, the achievable rate of MS$_k$, $\forall k \in [1, K]$, without the knowledge if it would directly transmit to a particular BS or indirectly transmits to a particular BS via a certain RS, can be expressed as

$$c_k = \sum_{n=1}^{N} x_{kn} c_{kn}^{\text{MS}-\text{BS}} + \sum_{m=1}^{M}\sum_{n=1}^{N} y_{km} z_{mn}^{(k)} c_{kmn}^{\text{MS}-\text{RS}-\text{BS}} \,. \tag{11}$$

Note that since an MS is either directly connected with a BS or indirectly connected with a BS via an RS, in (11), the value of only one x_{kn} or the values of a pair of $(y_{km}, z_{mn}^{(k)}$ would be 1 and the values of all other binary variables would be 0.

III. CAPACITY MAXIMIZATION PROBLEM

In this work, we want to solve the following joint optimization problem. Given the candidate locations of BSs and RSs, a number of MSs, maximum power budget of MSs and RSs, and minimum data rate demand of MSs, find the optimal power allocations $\{P_k\}$, $\{Q_{km}\}$, $\{F_{km}\}$, optimal location variables b_n, r_m, and optimal location allocation variables $\{x_k\}$, $\{y_{km}\}$, and $\left\{z_{mn}^{(k)}\right\}$ such that the sum-capacity of the system is maximized while the minimum data rate demand $\left\{r_k^{\min}\right\}$ of each MS is met.

Let $P_{\max}$ and $F_{\max}$ be the maximum transmit power budget of MSs and RSs. Using the notations defined in the previous section, the above optimization problem can be mathematically expressed as follows.

$$\texttt{Maximize} \quad \sum_{k=1}^{K} c_k \tag{12a}$$

$$\texttt{Subject to} \quad c_k \geq c_k^{\min}, \quad \forall k \tag{12b}$$

$$\sum_{n=1}^{N} x_{kn} + \sum_{m=1}^{M} y_{km} = 1, \quad \forall k \tag{12c}$$

$$x_{kn} y_{km} = 0, \quad \forall k, n, m \tag{12d}$$

$$\sum_{n=1}^{N} z_{mn}^{(k)} = r_m, \quad \forall m, k \tag{12e}$$

$$x_{kn} \leq b_n, \quad \forall k, n \tag{12f}$$

$$y_{km} \leq r_m, \quad \forall k, m \tag{12g}$$

$$z_{mn}^{(k)} \leq b_n, \quad \forall k, n, m \tag{12h}$$

$$\sum_{n=1}^{N} b_n = \tilde{N} \tag{12i}$$

$$\sum_{m=1}^{M} r_m = \tilde{M} \tag{12j}$$

$$0 \leq P_{kn} \leq P_{\max}, \quad \forall k, n \tag{12k}$$

$$0 \leq Q_{km} \leq P_{\max}, \quad \forall k, m \tag{12l}$$

$$0 \leq F_{km}^{(n)} \leq F_{\max}, \quad \forall k, n, m \tag{12m}$$

$$x_{kn} \in \{0, 1\}, \quad \forall k, n \tag{12n}$$

$$y_{km} \in \{0, 1\}, \quad \forall k, m \tag{12o}$$

$$z_{mn}^{(k)} \in \{0, 1\}, \quad \forall k, n, m \tag{12p}$$

$$b_n \in \{0, 1\}, \quad r_m \in \{0, 1\}, \quad \forall n, m \tag{12q}$$

The objective function (12a) maximizes the sum-capacity of the system. Constraint (12b) ensures that the data transmission rate of each MS is not smaller than its minimum rate requirement. Constraint (12c) along with the constraint (12d) states that an MS is either directly connected with a BS or via a single RS, whereas constraint (12e) makes sure that an RS assisting a particular MS can only be connected to a single BS; also if the RS is not deployed in a candidate location, it cannot be associated with a BS. Constraints (12f), (12g), and (12h) ensure that no MS is assigned with a BS that is not deployed in a BS CL, no MS si assigned with an RS that is not deployed in an RS CL, and no RS is assigned with a BS that is not deployed in a BS CL, respectively. Constraints (12i) and (12j) state that the total numbers of BSs and RSs that need to be deployed are $\tilde{N}$ and $\tilde{M}$, respectively, where $\tilde{N}$ and $\tilde{M}$ are chosen by the network designers. The non-negativity of the power allocation variables and the maximum power budget constraints of MSs and RSs are captured by constraints (12k), (12l), and (12m). Finally, constraints (12n), (12o), (12p), and (12q) satisfy the conditions that MS-BS, MS-RS, RS-BS assignment variables, and BS and RS location variables are binary.

The optimization problem described in (12) is a mixed integer nonlinear programming (MINLP) problem, which is NP-hard in general, due to the discrete nature of the MS-BS, MS-RS, RS-BS assignment variables, BS and RS location variables, and the continuous nature of the power allocation variables and all the constraints associated with both those discrete and continuous variables. The optimal solution of (12) can be obtained by exhaustive search algorithm which is computationally intractable due to its exponential complexity with respect to the number of MSs and RSs. A popular approach to solve MINLP problems is to convert them into mixed integer linear programming (MILP) problems which are computationally less intractable. In this paper, we use the MATLAB based commercial software package TOMLAB [8], which uses branch-and-bound algorithm, to get the near-optimal solutions of the capacity maximization problem.

It should be noted that our objective in this work is not to analyze the capacity maximization problem (12). Instead, we use the results of (12) for different combinations of the numbers of BS and RS to be optimally placed into CLs to investigate the energy-efficiency trade-offs between BS and RS. In the next section, we provide an iterative algorithm that would be used to investigate this trade-off.

IV. ITERATIVE ALGORITHM

In out iterative algorithm, first, we consider a BS-only deployment architecture. In other words, first, we select $0 \leq \tilde{N} \leq N$, the number of BSs to be optimally placed into their candidate locations and $\tilde{M} = 0$, the number of RSs to be optimally placed into their candidate locations, and solve the capacity maximization problem to find the maximum sum-capacity $c_{\max}$ under this deployment architecture. Next, we reduce the number of BSs to be optimally placed into their candidate locations by 1, i.e. $\tilde{N} = \tilde{N} - 1$ and increase the number of RSs to be deployed optimally into their candidate locations by 1, i.e. $\tilde{M} = \tilde{M} + 1$ and solve the capacity maximization problem (12). If the sum-capacity, c_m, obtained by this combination of BSs and RS is close to $c_{\max}$, we conclude that the maximum sum-capacity, $c_{\max}$, that can be obtained by $\tilde{N}$ BSs can also be obtained by $\tilde{N} - 1$ BSs and

TABLE I
ITERATIVE ALGORITHM

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Require: $N$, $M$, $K$, $c_k^{\min}$, $P_{\max}$, $F_{\max}$.
 1: set $0 \le \tilde{N} \le N$ and $\tilde{M} = 0$.
 2: solve the capacity maximization problem (12) to find $c_{\max}$.
 3: while $\tilde{N} > 1$ do
 4:     $\tilde{N} = \tilde{N} - 1$
 5:     while $\tilde{M} \le M$ do
 6:         repeat
 7:             $\tilde{M} = \tilde{M} + 1$
 8:             solve the capacity maximization problem (12) to find $c_m$.
 9:         until $(|c_m - c_{\max}|)/c_{\max} \le \delta$
10:         calculate the total transmitted power by MSs and RSs.
11:     end while
12: end while
```

1 RS. Otherwise, each time, we keep increasing the number of RSs to be deployed optimally into their candidate locations by 1 and solve the capacity maximization problem (12) until c_m is close to $c_{\max}$ with some accuracy δ. We continue this process of reducing $\tilde{N}$ by 1 and finding the number of RSs $\tilde{M}$ that gives the same maximum sum-capacity $c_{\max}$ until $\tilde{N} = 1$ and $\tilde{M} \le M$. The condition $\tilde{N} = 1$ and $\tilde{M} \le M$ ensures that there should be at least 1 BS in the system and the number of RSs to be deployed optimally cannot exceed the number of candidate locations to deploy them. This iterative algorithm gives a number of optimal combinations of BSs and RSs to achieve a maximum sum-capacity $c_{\max}$. For each combination or BSs and RSs, we calculate the total MS and RS transmission powers which gives us energy-efficiency trade-offs between BS and RS. We summarize this iterative algorithm in Table I.

V. NUMERICAL RESULTS

In this section, we provide some numerical results to show the energy-efficiency trade-offs between BS and RS. In our simulation model, we consider a square service area of size 1000m×1000m. There are $K = 50$ MSs uniformly distributed within this area. The numbers of fixed candidate locations for BSs and RSs are $N = 20$ and $M = 40$, respectively. The maximum power budget for MSs and RSs are $P_{\max} = 0.1$ W and $F_{\max} = 0.5$ W, respectively. The path loss exponent is 3. It is assumed that all the receivers at RSs and BSs are subject to Additive White Gaussian Noise (AWGN) with zero mean and unit variance. We assume flat Rayleigh fading channel among all the links. This is ensurd by generating the channel coefficients as circularly symmetric AWGN with zero mean and unit variance. Minimum traffic demand of MSs are uniformly generated in $(0, 1]$. For the iterative algorithm, we use $\tilde{N} = 10$ and $\delta = 0.01$. In line 2 and line 8 of the iterative algorithm, the results of the capacity maximization problem are averaged over 100 channel instances. The location of MSs are changed over each instance. However, the candidate locations of BSs and RSs and minimum data demand of each MS are fixed over all channel instances. The optimal locations of BSs and RSs are chosen as the maximum number of times the BS and RS locations are chosen over 100 channel realizations. Total power consumption are determined by implementing the capacity maximization problem after finding the optimal locations of BSs and RSs. We use the commercial MATLAB software TOMLAB to solve the capacity maximization problem (12).

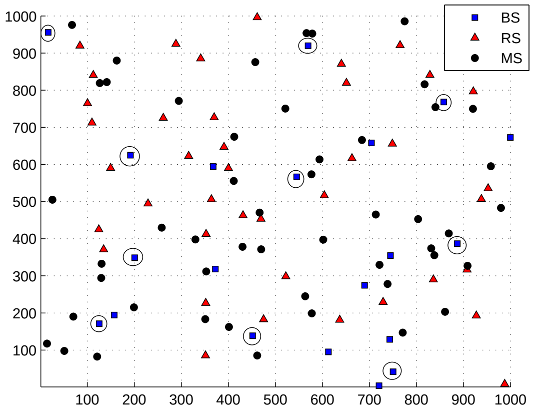

Fig. 1. Locations of MSs, candidate locations of BSs and RSs, and optimally chosen $\tilde{N} = 10$ BS deployment locations; here $K = 50$, $N = 20$, and $M = 40$.

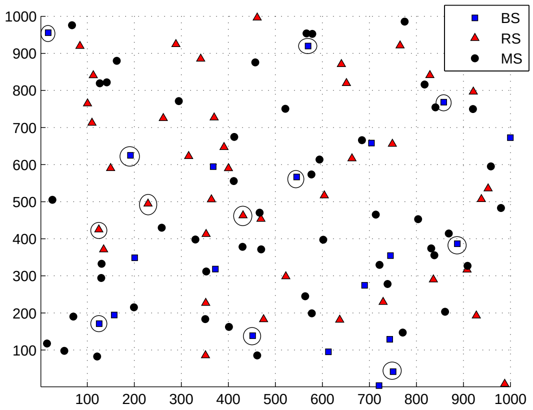

Fig. 2. Locations of MSs, candidate locations of BSs and RSs, optimally chosen $\tilde{N} = 9$ BS deployment locations, and optimally chosen $\tilde{M} = 3$ RS deployment locations; here $K = 50$, $N = 20$, $M = 40$, $\tilde{M} = 0$.

Fig 1 shows the network deployment scenario for 50 MSs, 20 BS candidate positions, 40 RS candidate positions. This is the BS-only architecture. After running the iterative algorithm with $\tilde{N} = 10$ and $\tilde{M} = 0$, we get the 10 optimal locations for BS deployment which are are shown in circles.

Next, in Fig 2, we show the optimal locations of 9 BSs and 3 RSs after reducing $\tilde{N} = 10 - 1 = 9$. Note that after reducing $\tilde{N}$ by 1, in order to achive the same sum-capacity that was obtained by optimally deploying 10 BSs, our algroithm optimally selects 9 BS deployment positions and 3 RS deployment positions.

Finally, in Fig 3, we show the impact of replacing BSs by RSs on the selection of optimal deployment locations for both BSs and RSs, and the total power consumption to achieve the same the sum capacity that was obtained with $\tilde{N} = 10$. To explain the results provided in Fig 3, let us look at an

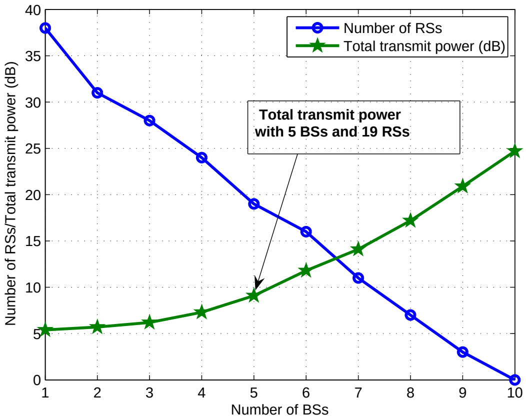

Fig. 3. Energy-efficiency trade-offs of BS and RS in terms of total transmit power

example. The maximum sum capacity achieved with 10 BSs was 8.161 and 25 dB of total MS transmit power was needed to achieve this capacity. When we reduce the number of BSs to 5, it required 5 BSs, 19 RSs and 9 dB of total MS and RS transmit power to achieve the capacity of 8.161. Now, it is clear from Fig 3 that replacing BSs with RSs leads to significant reduction in total transmit power. By reducing the number of BSs from 10 to 2, the total transmit power can be reduced from 25 dB to almost 5 dB. However, looking at the slope of the total transmit power curve with respect to number of BSs, one can easily conclude that the gain in power saving shows a diminishing trend as the number of BSs becomes smaller. It can also be observed that no further gain in the reduction of total transmit power is achieved after $\tilde{N} = 2$. One might suspect intuitively that if we start with a larger number of BSs, and hence a larger sum-capacity requirement, we might see a minimum point in total transmit power after which an increase in total transmit power could happen with further reduction of the number of BSs. This result could be explained as follows. As the number of BSs decreases, less number of MSs will have direct communication with BSs. Moreover, due to larger BS-RS and RS-MS distances, both MSs and RSs will require more transmit powers for succesfull communications.

VI. CONCLUSION

For a WRN, we have studied the optimal placement of BSs and RSs into a predefined set of candidate locations, optimal assignment of MSs with optimally placed BSs and RSs, and optimal allocation of MS and RS powers to investigate the energy-efficiency trade-offs between BS and RS. We have provided an iterative algorithm that determines the optimal combination of BSs and RSs that is required to be placed optimally at the candidate locations in order to achieve a certain maximum sum-capacity that can be achieved with a BS-only architecture. We have shown by simulation that significant reduction in total transmit power can be obtained by replacing BSs with RSs. However, this gain in the reduction of total transmit power diminishes as the number of BSs becomes smaller which makes the BS-RS and RS-MS distances too large for energy efficient communications.

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