

The 15 Puzzle: How it Drove the World Crazy

by Jerry Slocum and Dic Sonneveld

The Slocum Puzzle Foundation, 2006.
Hardcover, 144 pages, \$30.00. ISBN 1890980153

Some years ago, I wrote an article about the 15 puzzle that began, “In the 1870’s the impish puzzlemaker Sam Loyd caused quite a stir in the United States, Britain, and Europe with his now-famous 15-puzzle” [1]. I’ve always been pleased with myself for managing to slip the word “impish” into a published mathematics article. As it turns out, this devilish descriptor was the most accurate part of that sentence, as Slocum and Sonneveld document in their new book *The 15 Puzzle: How it Drove the World Crazy*. The 15 puzzle did once cause an intense craze that spread like wildfire across America and overseas, and it is indeed famous to this day. However, the initial fad did not occur until 1880, and Sam Loyd had nothing to do with it until eleven years later, when he started to claim in print that he had invented the puzzle. Through meticulous research using primary sources, Slocum and Sonneveld not only expose Sam Loyd’s fraudulent claims, but also argue convincingly that the actual inventor was Noyes Chapman, the postmaster of Canastota, New York.

The 15 puzzle is a sliding block puzzle consisting of 15 numbered square blocks placed inside a frame large enough to accommodate 16 blocks in a 4 by 4 grid (see Figure ***). The empty space allows the solver to slide any of the adjacent blocks into the space. Given a starting configuration, the puzzle is to reach some specified target configuration via a sequence of such moves. The instructions written on the cover of the original puzzle read, “Place the Blocks in the Box irregularly, then move until in regular order” (p. 8). It did not specify exactly what is meant by “regular order,” but it did contain a picture of the blocks arranged as shown in Figure ***. Later posers of the puzzle were more careful to explicitly state this as the target state. For the purposes of our discussion, let us call this the *canonical* state. Early fans of the puzzle discovered that they could often reach a configuration that differed from the canonical state only in that the 14 and 15 were swapped, but try as they might, they could not complete the puzzle. Thus, the problem of solving the puzzle from this starting state became the standard challenge, which we will call the *14-15 puzzle*.

Beginning in 1880, numerous cash prizes of up to \$1000 American (a princely sum in those days) were offered for the solution of the 14-15 puzzle. There have been numerous published accounts of people spending endless hours engrossed in the puzzle, but nobody ever successfully claimed these prizes. An intriguing mathematical fact about the 15 puzzle is that for exactly half of the 16! possible initial configurations, the puzzle is impossible to solve. It should come as no surprise that the 14-15 puzzle starts from one of these impossible configurations.

The set of solvable configurations can be easily described using the theory of even and odd permutations. First, it should be clear that the set of all solvable configurations are precisely those that can be reached starting from the canonical state, since each move is reversible. Second, let us imagine the blank space to contain a block that we will call the “blank block.” Then each move consists of swapping the blank with one of the blocks that is adjacent to it horizontally or vertically.

Third, let us focus on the set of attainable configurations where the blank lies in the lower right corner. Then the question is which of the $15!$ permutations of 16 blocks that fix the blank can be attained, starting from the canonical configuration.

Let us consider an arbitrary sequence of moves that returns the blank to its initial position. Since each move swaps the blank with another block, we may write the resulting permutation as a product of transpositions, one per move. Now color the spaces of the board white and black in a checkerboard pattern, with the lower right corner colored black. Since the color of the square occupied by the blank switches with each move and it starts and ends on black, it must make an even number of moves, so the resulting permutation may be written as a product of an *even* number of transpositions. Now recall the well-known fact that every permutation can be written as a product of either an even or odd number of transpositions, but not both. Then the argument we just made shows that no odd permutation that fixes the blank is attainable. It is also the case that every even permutation that fixes the blank *is* attainable, although that result is far less obvious. To complete the picture, a configuration of the puzzle with the blank in an arbitrary position is obtainable from the canonical configuration if and only if either the blank is on a black square and the permutation is even, or the blank is on a white square and the permutation is odd. It is against this mathematical backdrop that the remarkable history of the 15 puzzle plays itself out.

Slocum and Sonneveld pick up their tale with the first commercial production of the puzzle beginning in December 1879, and proceed to trace its explosive spread over the first half of 1880. In December 1879, a crude version of the 15 puzzle made its way into the hands of Matthias Rice, a Boston woodworker who began manufacturing the puzzles in his shop. After some effort, he got one of the leading toy dealers in Boston to carry them. They became an immediate commercial success, attracting many other manufacturers to jump into the fray before the beginning of March.

The craze spread from Boston to New York, all along the East Coast and across America, and to Canada, Europe and beyond. It cut across all sections of American life, and references to the 15 puzzle appeared in many venues of American pop culture. Stage productions, music and poems featured the puzzle, and numerous newspaper and magazine articles appeared, discussing the hold the puzzle was taking on the nation's collective imagination. Most of these focused on people's obsession with the 14-15 puzzle, and there was a great deal of public argument over whether or not it could be solved. Many of the articles contain (likely exaggerated) reports of the puzzle's role in filling the insane asylums, and many sound tongue-in-cheek alarms about the supposed deleterious effect the puzzle was having on the fabric of society. On February 17, 1880, the *Rochester Democrat and Chronicle* published an article titled, " '15': The Diabolical Invention of Some Enemy of Mankind." (p. 22) The New York Times followed up with a whimsical story on March 22 that opined, "No pestilence has ever visited this or any other country which has spread with the awful celerity of what is popularly called the 'Fifteen Puzzle.' ...it now threatens our free institutions, inasmuch as from every town and hamlet there is coming up a cry for a 'strong man' who will stamp out this terrible puzzle at any cost of Constitution or freedom" (pp 53-54). Much of the book's narrative is driven by these numerous newspaper accounts that deal with the popularity and addictiveness of the puzzle, and some of which dig into the murky history behind its invention.

The authors make very clever use of these articles in constructing a series of tables (pp. 58-60) that provide an illustrated timeline of the progress of the craze across the country and the world. For each of a representative sample of 24 cities spread geographically from Boston to Los Angeles, a bar connects the dates of the first and last articles about the puzzle in the newspapers of that city, along with a special marker if the article listed the craze as having just started or just ended.

It seems that the craze hit its peak in most American cities in February or March of 1880, and started to abate by April. The authors conclude that in most parts of the United States, the craze had ended by May. A similar chart shows the puzzle spreading through Europe in April and May, and even as far as Australia and New Zealand.

Throughout the detailed documentation of the 15 puzzle craze of 1880, there are many moments of comic relief. A dentist from Worcester, Massachusetts offered a \$25 set of false teeth for a solution to the 14-15 puzzle, which he later augmented by an additional \$100 cash prize (p. 16). Shortly thereafter, another citizen of Worcester placed an ad that read, “I have a cat that possesses a fine set of teeth, and I will give to any one the Entire Set and \$100.00 (One Hundred Dollars), who will transpose two of her teeth and then put them back into their original positions and not hurt the cat. Operators to take all the chances of getting bitten. Dentists are not allowed to compete” (p. 111).

A large portion of the book is devoted to documenting the initial 15 puzzle craze. This is interesting not only for the entertainment value of the attendant witticisms, but also for the insight it lends into the anatomy of a large-scale fast-moving fad. It also establishes definitively that the craze occurred in 1880, a key piece of evidence in support of the book’s main thesis — that the universally accepted belief that Sam Loyd invented the 15 puzzle is actually a myth.

Slocum and Sonneveld second Martin Gardner’s description of Sam Loyd as “America’s Greatest Puzzlist,” based on the work he actually did (p. 75). However, they also reveal a man who had a habit of trying to augment his legitimate fame by claiming credit for the work of others, using, for example, inventions of the English puzzlist Henry Dudeney without credit, and by claiming that he invented the game *Parcheesi* and a popular dexterity puzzle called *Pigs in Clover*. However, his most successful intellectual theft was the 15 puzzle.

From 1891 until his death in 1911, Sam Loyd mounted an enduring disinformation campaign to take credit for the invention of this puzzle. His stories were not all consistent, and indeed it seems that part of his strategy may have been to muddy the waters. For instance, in various writings and interviews he claimed to have invented the puzzle at different times from 1872 to 1878. He was so successful in this undertaking that Slocum and Sonneveld found not a single article that questioned Loyd’s claims, and every obituary that they found for him stated that he had invented the 15 puzzle. For 115 years, Loyd’s phony claims were accepted as established fact, until these authors painstakingly sorted through mountains of primary source material and made Swiss cheese of them.

Having debunked the myth that Loyd invented the 15 puzzle, the authors then set out to discover the actual inventor. Through a tour de force of forensic social archaeology, Slocum and Sonneveld found that the puzzle was invented by Noyes Chapman, and traced the puzzle’s path from Mr. Chapman in upstate New York to Matthias Rice in Boston. No step along this path is proven beyond a doubt, and it seems unlikely that it could be proven now after more than 125 years, but the evidence that the authors dug up strikes me as very compelling. They even located a U.S. patent application filed by Mr. Chapman on February 21, 1880, but found no explanation of why the patent was rejected. Perhaps, they conjecture, Chapman’s invention was judged too similar to a patent granted to Ernest Kinsey in 1878 for a different sliding block puzzle. Ironically, it is precisely the slight difference in the mechanical design of Chapman’s puzzle that leads to the mathematical impossibility that fueled its astounding success.

The book catalogs some variants of the 15 puzzle, such as the challenge of sliding the blocks to form a magic square, and to do so in the minimum number of moves. It also explains many

solutions that involve cheating in some fashion, such as rotating the box 90 degrees, or leaving the blank in the upper left corner, both of which exploit the ambiguity in the original instructions of what constitutes “regular order.”

The book also gives a fair amount of attention to the mathematical literature on the puzzle. Here too, historical curiosities arise. A pair of articles by Wm. Woolsey Johnson and William E. Story in the *American Journal of Mathematics* (December, 1879) have been widely cited in the literature as the first to characterize exactly which configurations of the 15 puzzle can be reached starting from the canonical configuration – Johnson proved that all odd permutations are impossible [2], while Story demonstrated that all even ones are attainable [5]. Slocum and Sonneveld became suspicious about the supposed publication date because an attached note by the editors refers to the ongoing 15 puzzle craze, which would have been inconsistent with the preponderance of documentary evidence, and Johnson’s article refers to a newspaper account that turns out to have appeared on March 5, 1880. The authors discovered that the publication of this issue of the journal had been delayed until at least late March 1880, and the issue was not received by the Library of Congress until April 17 (pp. 66-67). The false date, together with Sam Loyd’s machinations, caused later writers, myself included, to falsely conclude that the 15 puzzle craze had occurred in the 1870s.

Meanwhile, in a newspaper article in the *Hamburgischer Correspondent* of April 6, 1880 the German mathematician Hermann Schubert published a proof in German of the impossibility of obtaining odd permutations [3]. At this time it is unknown whether Johnson’s or Schubert’s proof was published first, let alone who submitted first. Schubert’s proof went overlooked by later scholars, possibly because it did not appear in a peer-reviewed journal. Mathematicians are not used to looking for original research in newspapers!

Johnson’s and Schubert’s proofs are explained in a short section guest-authored by Dick Hess (pp. 117-119). He does a very good job of digesting the arcane language used in the original proofs and presenting them in a fashion that the layman should be able to understand. Thereby, he does a service to our discipline in demonstrating to a broad audience how careful mathematical reasoning can accomplish what experimentation cannot, in this case resolving the dispute over the solvability of the 15 puzzle that had raged in numerous newspaper editorials as part of a society-wide debate. He also gives another glimpse of the power of mathematical abstraction by explaining Richard Wilson’s generalization of the 15 puzzle to a game of swapping labels between adjacent vertices in arbitrary graphs, and his remarkable theorem characterizing which configurations are solvable [6].

Unfortunately, Hess also makes several small mathematical missteps. In his exposition of Johnson’s proof, he defines a permutation of n objects as being even or odd according to the parity of the number of cycles in its cycle representation. Luckily, this turns out to coincide with the usual definition when n is even, as it is for the case under consideration, namely the 16 spaces in the 15 puzzle. But when n is odd, this definition conflicts with the usual one. In explaining Wilson’s result, he omits an important hypothesis of the theorem, namely that the graph under consideration contain no cut vertex. He also dismisses papers by Edward Spitznagel [4] and myself [1] as “essentially following Johnson’s approach,” even though both of these papers actually give alternate proofs of Story’s result that all even permutations are attainable. Much of the mathematical literature on the 15 puzzle obscures the distinction between proving that no odd permutation is attainable and proving that all even permutations are attainable, so I was sad to see that this book did not do more to clarify that point.

In addition to the superb historical research conveyed in the text, this book contains dozens of

beautiful illustrations, including photos of many different versions of the 15 puzzle. These, and an abundance of humor, make it a pleasure to flip through for the casual reader, while the compelling historical account rewards those who read it cover to cover. For those interested in harnessing the power of fads, it provides a valuable detailed case study. But most important is the unclocking of Sam Loyd's wildly and enduringly successful hoax. Today's citizenry would be wise to take this episode as a warning against the "he said, she said" reporting that so often poses as journalism. It was a basic lack of fact-checking by journalists starting in 1891 that allowed Loyd's deception to get off the ground, followed by an echo chamber effect that elevated it to the status of accepted history. Kudos to Slocum and Sonneveld for setting the record straight.

References

- [1] A.F. Archer, "A modern treatment of the 15 puzzle," *Amer. Math. Monthly* 106 (1999) 793-799.
- [2] W.W. Johnson, "Notes on the '15' puzzle I," *Amer. J. Math.* 2 (1879) 397-399.
- [3] H. Schubert, "The Boss puzzle," *Hamburgischer Correspondent*, 6 April 1880.
- [4] E.L. Spitznagel, Jr., "A new look at the fifteen puzzle," *Math. Mag.* 40 (1967) 171-174.
- [5] W.E. Story, "Notes on the '15' puzzle II," *Amer. J. Math.* 2 (1879) 399-404.
- [6] R.M. Wilson, "Graph puzzles, homotopy, and the alternating group," *J. Combin. Theory* (Series B) 16 (1974) 86-96.

Aaron Archer
AT&T Shannon Research Laboratory
180 Park Avenue
Florham Park, NJ 07932
USA
e-mail: aarcher@research.att.com