# Interleaver design for Turbo codes

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*Abstract*— **A new interleaver design to improve the performance of the Turbo codes is presented here. Two criteria are considered in the design of the interleaver, i.e., distance spectrum properties of the code and the reduction of the correlation between the extrinsic information and the input data at each iteration.**

## I. Introduction

TURBO codes[1] have an impressive near Shannon limit error correcting performance. This superior performance of Turbo codes compared to convolutional codes is only achievable when the length of the interleaver is very large, on the order of several thousand bits. For large block size interleaver, most random interleavers perform. For short block length Turbo codes, a deterministic or semi-random interleaver can be more effective than random interleavers.

An interleaver $\pi$ is a permutation $i \mapsto \pi(i)$ that maps a data sequence of $N$ input symbols into the same sequence with a new order. An S-random [2] interleaver is a semi-random interleaver that performs better than most random interleavers. Each randomly selected integer is compared to S previously selected random integers. If the distance between this integer and previously selected random integers is greater than S, then it is selected. Otherwise, a new random integer will be chosen and the above condition is tested. This process repeats until all $N$ distinct integers have been selected in this random order. Computer simulation results have shown that if $S \le \sqrt{\frac{N}{2}}$, then this process will converge [2]. This interleaver design assures that the short cycle events are avoided. Short cycle event occurs when two bits are close to each other before and after interleaving.

## II. 2-step S-random interleaver design

A new interleaver design, 2-step S-random interleaver, is presented here based on the S-random interleaver. The 2-step S-random interleaver is designed under the constraint to increase the minimum effective free distance of the Turbo code while decreasing the correlation properties between the information input data sequence and extrinsic information. The criterion used in the second step of the design to decrease this correlation is based on the revised version of iterative decoding suitability (IDS) condition that is described in [3-4].

**Step 1**: Design an interleaver of length N that for all integer values of $i$ choose its corresponding $\pi(i)$ without repetition and having the following two properties:

- For all $i$ and $j$ if $|i-j| \le S_1 \Longrightarrow |\pi(i)-\pi(j)| > S_1$.
- For all $i$ and $\pi(i)$, then $|i-\pi(i)| > S_2$.

where $S_1$ and $S_2$ are pre-determined.

**Step 2**: Choose a pre-determined minimum effective free distance code [5] $d_{min1}$. Also define $w_{det}$ as the maximum input data sequence weight. Find all input data sequences of length N and weight $w_l \le w_{det}$ such that $d_{min,w_l} < d_{min1}$. Suppose one of these input data sequences of length N and weight $w_1$ has the following non-zero interleaver pairs $(i_1,\pi(i_1)),(i_2,\pi(i_2)),\ldots,(i_{w_1},\pi(i_{w_1}))$ with $d_{min,w_1} < d_{min1}$. Compute $IDS_{(new)}$ based on [6] for the original interleaver that was designed in step 1. Set $j = i_1+1$ and find the pair $(j,\pi(j))$. Interchange the interleaver pairs $(i_1,\pi(i_1))$ and $(j,\pi(j))$ to create a new interleaver, i.e., $(i_1,\pi(j))$ and $(j,\pi(i_1))$. Compute the new IDS, $IDS'_{(new)}$, based on the new interleaver design. if $IDS'_{(new)} \le IDS_{(new)}$, the new interleaver design will replace the previous one. Otherwise, set $j = j+1$ and continue this search until a new interleaver with smaller $IDS'_{(new)}$ is found. Repeat this operation for all input data sequences with a minimum weight of $w_{det}$ that have $d_{min} < d_{min1}$. After this operation completes, go back to step 2 again and find all input data sequences of weight $w_{det}$ or less with $d_{min} < d_{min1}$ for the new interleaver. Continue this step untill it converges and there is not any input data sequence of weight $w_{det}$ or less with $d_{min} < d_{min1}$. Obviously if $d_{min1}$ is chosen a large value, the second step may never converge. In this case, a smaller value for $d_{min1}$ should be selected. The following theorem describes a technique to design an interleaver based on step 1.

**Theorem 1:** Let $\alpha \in \mathbf{N}$ be a natural number such that $gcd(\alpha,N)=1$ and $\alpha-1$ divides $N$. Then there is a permutation $\pi \in S_N$ satisfying (i) and (ii) with $S_1 := min(\alpha, \frac{N}{\alpha+1})$ and $S_2 := \lfloor \frac{\alpha-1}{2} \rfloor$. Let $\beta := \lfloor \frac{\alpha-1}{2} \rfloor$ and define $\pi : \{1,\ldots,N\} \to \{1,\ldots,N\}$ by $\pi(i) := \alpha \cdot i + \beta$, where $\pi(i)$ has to be interpreted as the number $\pi(i) \in \{1,\ldots,N\}$ that is congruent to $\alpha \cdot i + \beta$ modulo $N$.

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